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THE COOLING OF FIBRES IN THE FORMATION PROCESS

D. E. BOURNE and H. DIXON

Departments of Applied Mathematics and Computing Sciences, University of Sheffield, Sheffield S10 2TN, England

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Abstract—In the glass and polymer industries, fibres are manufactured by extruding hot material through a circular orifice to form a continuous filament. This paper is concerned with predicting the temperature of the fibre as a function of distance from the orifice.

A simple model is examined wherein the fibre is regarded as a continuous infinite circular cylinder issuing into a fluid of infinite extent. The boundary layer equations, in conjunction with an equation governing the rate of supply of heat to the fluid from the fibre, are solved using the von Kármán–Pohlhausen method.

An important difficulty in this problem is that the fluid properties vary greatly over the region of interest, due to the large temperature gradients which occur in practice. To circumvent this, a method of averaging is devised and this leads to fairly close agreement with experimental results.

NOMENCLATURE

a,	radius of fibre;
a_1, a_2, a_3,	coefficients in series expansion (B.1);
C_S,	specific heat of fibre;
C_p,	specific heat of fluid;
I,	integral defined by equation (28);
k,	thermal conductivity;
p, q,	coefficients in temperature profile (A.1);
Q,	rate of heat transfer per unit length of fibre;
r,	distance from the axis of the fibre;
T,	temperature of fluid;
T_S,	temperature of fibre;
T_∞,	ambient temperature of fluid;
T_0,	temperature of the fibre at the orifice;
$T_S^{(1)}, T_S^{(2)}, \ldots$	first, second, ... approximations to T_S;
U,	speed of the fibre;
u, v,	axial and radial fluid velocity components;
x,	axial coordinate;
x_f,	axial distance at which the fibre temperature is T_f;
y,	distance from the surface of the fibre.

Greek symbols

α,	parameter in boundary layer velocity profile, equation (10);
β,	parameter in boundary layer temperature profile, equation (16);
γ,	Euler's constant;
δ,	momentum boundary layer thickness;
δ_T,	thermal boundary layer thickness;
κ,	thermal diffusivity;
η,	ratio of specific heats, per unit volume, of the fluid and the fibre;
ρ,	density of the fluid;
ρ_S,	density of the fibre;
σ,	Prandtl number $[\nu/\kappa]$;
θ,	$T - T_\infty$;
θ_S,	$T_S - T_\infty$;
ν,	kinematic viscosity.

1. INTRODUCTION

In the process of manufacturing glass or polymer fibres, filaments of hot materials are drawn through orifices and cool as they pass through the surrounding environment. In a recent paper, Bourne and Elliston [1] analysed a simple model of this process wherein a fibre is treated as a continuous infinite circular cylinder issuing from a circular orifice and penetrating a fluid of infinite extent. The object of their paper was to provide a method for calculating the average rate of heat transfer from a given length of fibre, and this was accomplished on the basis of the assumption that the non-uniform fibre temperatures experienced in practice may be replaced by uniform average values. Effectively, this procedure supposes that the heat capacity of the fibre is infinite and naturally gives no direct information about how the fibre temperature varies with distance from the orifice.

The purpose of the present contribution is to show how allowance may be made for the finite heat capacity of the fibre. Attention will be given mainly to calculating the fibre temperature as a function of distance from the orifice, but the local heat transfer coefficient can also be calculated quite easily. The problem is approached through the laminar boundary layer equations, and an approximate solution is obtained by means of the von Kármán–Pohlhausen integral technique.

A major source of difficulty is that the temperature range in the actual manufacturing process is so wide that some of the fluid properties are far from uniform. In the model proposed here, these non-uniform values are replaced throughout by uniform average values, but close attention is then given to devising a satisfactory method whereby the average values may be chosen.

Several papers [1–6] have appeared recently on heat transfer in boundary layers on continuous moving surfaces. The only one of these which deals directly with finite heat capacity effects and a fluid of arbitrary Prandtl number is that of Erickson *et al.* [3]. They considered a hot flat sheet issuing from a slot into a fluid and calculated the rate of heat transfer and its surface temperature. This is the two-dimensional counterpart of the problem under consideration here. Fundamental to the papers [1–6] on heat transfer are those of Sakiadis [7–9] in which the momentum boundary layer equations for flows over flat and cylindrical surfaces are set up and solved. Once again we shall have recourse to Sakiadis's work.

Some experimental data on the temperature distribution along a drawn fibre has been provided by Alderson *et al.* [10] and by Arridge and Prior [11]. It will be shown that the proposed theoretical model gives results in fairly close agreement with those obtained experimentally.

2. THE BASIC EQUATIONS

The physical model is that of an endless circular fibre issuing steadily, with speed U, from an orifice into a semi-infinite fluid medium (Fig. 1). At large distances from the fibre, the fluid is at rest and at a uniform temperature T_∞.

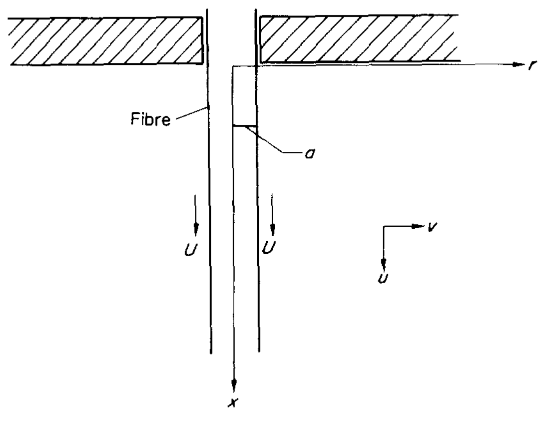

FIG. 1. Endless circular fibre drawn steadily downwards through a circular orifice.

We take coordinates x and r which measure distance along the axis of the fibre from the orifice and distance from the axis, respectively;

the corresponding fluid velocity components are u and v, and the fluid temperature is denoted by T.

On the basis of the boundary layer approximations, which Bourne and Elliston [1] showed are applicable in the present circumstances, the equations of motion and the energy equation reduce to

$$r\frac{\partial u}{\partial x} + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right); \tag{3}$$

ν, κ are, respectively, the kinematic viscosity and thermal diffusivity of the fluid. In the derivation of these equations, spatial variations of the fluid properties have been neglected; we shall discuss this approximation later. The boundary conditions are:

$$u = U, \quad v = 0, \quad T = T_S(x) \text{ at } r = a, \tag{4}$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty \quad \text{as } r \to \infty, \tag{5}$$

where a is the radius of the fibre and $T_S(x)$ is its surface temperature.

In the problem of Bourne and Elliston [1], T_S was assumed given and independent of x: the object here is to determine $T_S(x)$, and one further governing equation is therefore required. The additional equation arises from the fact that the fibre has a finite heat capacity and is steadily losing its heat to the surrounding fluid.

Consider the elementary cylindrical region $r \leqslant a$, $X \leqslant x \leqslant X + \delta X$. In a short time interval δt, the heat received by the fluid from the fibre passing through this region is, to first order,

$$-2\pi a k(\partial T/\partial y)_{y=0}\delta X \delta t, \tag{6}$$

where $y = r - a$ denotes distance from the surface of the fibre and k is the thermal conductivity of the fluid. The corresponding net loss of heat from the element by convection across its two bounding cross-sections is

$$-U\pi a^2 \rho_S C_S \mathrm{d}T_S/\mathrm{d}x\, \delta X \delta t, \tag{7}$$

where ρ_S and C_S, respectively, denote the density and specific heat of the fibre. Here we have assumed that T_S does not vary over the fibre cross-section: justification for this may be found in Glicksman [5]. It may also be shown that, under typical operating conditions, heat transfer in the axial direction by conduction along the fibre is very small compared with that due to convection and may be neglected. It thus follows from (6) and (7) that

$$k(\partial T/\partial y)_{y=0} = \tfrac{1}{2}Ua\rho_S C_S \mathrm{d}T_S/\mathrm{d}x, \tag{8}$$

and this is the final governing equation. We note that equation (8) requires modification by the inclusion of a heat source term if the material of the fibre undergoes a phase transition in the forming process. In the case of glass (with which we are mainly concerned), the material is in the liquid state throughout, but the behaviour of some polymers is more complicated and partial or total crystallisation may occur (Bawn [12]).

Because of the additional equation, another boundary condition is now required, and this is provided by assuming that the fibre temperature at the orifice (T_0) is known, giving

$$T_S(0) = T_0. \tag{9}$$

Using the von Kármán–Pohlhausen method, an approximate solution of equations (1) and (2) has already been given by Sakiadis [9] and has been put into a convenient form by Bourne and Elliston [1]. The velocity profile may be expressed as

$$\frac{u}{U} = 1 - \frac{1}{\alpha(x)}\log_e\left(1 + \frac{y}{a}\right) \quad \text{for} \quad y \leqslant \delta(x) \tag{10}$$

and

$$\frac{u}{U} = 0 \quad \text{for} \quad y \geqslant \delta(x) \tag{11}$$

where

$$\delta(x) = a(e^{\alpha} - 1) \tag{12}$$

is the boundary layer thickness; the parameter $\alpha(x)$ satisfies the implicit equation

$$\frac{2\nu x}{Ua^2} = \frac{e^{2\alpha} - 1}{\alpha} - Ei(2\alpha) + \log_e(2\alpha) + \gamma - 2, \tag{13}$$

where $\gamma = 0{\cdot}5772\ldots$ is Euler's constant.

In the next section we show that the temperature $T_S(x)$ of the fibre may also be determined by the von Kármán–Pohlhausen method.

3. ANALYSIS TO OBTAIN THE FIBRE TEMPERATURE PROFILE

Define

$$\theta(x, y) = T(x, y) - T_{\infty} \tag{14}$$

and

$$\theta_S(x) = T_S(x) - T_{\infty}. \tag{15}$$

We assume that the fluid temperature distribution may be expressed in the form

$$\frac{\theta}{\theta_S} = 1 - \frac{1}{\beta(x)}\log_e\left(1 + \frac{y}{a}\right) \quad \text{for} \quad y \leqslant \delta_T(x) \tag{16}$$

and

$$\frac{\theta}{\theta_S} = 0 \quad \text{for} \quad y \geqslant \delta_T(x), \tag{17}$$

where

$$\delta_T(x) = a(e^{\beta} - 1) \tag{18}$$

is the thickness of the thermal boundary layer. Justification for this choice of distribution, which reduces to that assumed by Bourne and Elliston [1] in the case when θ_S is independent of x, is given in the Appendix A.

To determine $T_S(x)$ and the parameter $\beta(x)$, we use the energy integral equation, viz.

$$\frac{d}{dx}\int_0^{\infty}(a + y)u\theta\,dy = -\kappa a\left(\frac{\partial\theta}{\partial y}\right)_{y=0}; \tag{19}$$

this may be derived from the boundary layer energy equation (3) by direct integration, use being made also of the integral of the continuity equation (1) to eliminate v.

At this stage it is convenient to restrict attention to cases where the thermal boundary layer contains the entire momentum boundary layer, so that $\delta \leqslant \delta_T$. This implies that the Prandtl number $\sigma \leqslant 1$; however, the case when $\sigma > 1$ differs from this only in subsequent details rather than in any fundamental way.

By substituting the velocity profile, (10) and (11), and the temperature profile, (16) and (17), into the integral equation (19), we obtain

$$\frac{d}{dx}\left\{\frac{\theta_S(x)}{\alpha\beta}\left[(\beta - \alpha + 1)e^{2\alpha} - (2\alpha\beta + \alpha + \beta + 1)\right]\right\} = \frac{4\kappa\theta_S(x)}{Ua^2\beta}. \tag{20}$$

The energy balance equation (8) may also be expressed in terms of $\theta_S(x)$ and $\beta(x)$ by substituting equations (14)–(16). Defining

$$\eta = \frac{\rho C_p}{\rho_S C_S}, \tag{21}$$

which is the ratio of the specific heats per unit volume of the fluid and the fibre, respectively, we find that

$$\frac{d\theta_S}{dx} = -\frac{2\kappa\eta}{Ua^2\beta}\theta_S. \tag{22}$$

Now, differentiation of (13) yields

$$\frac{dx}{d\alpha} = [(\alpha - 1)e^{2\alpha} + \alpha + 1]\frac{Ua^2}{2\nu\alpha^2}, \tag{23}$$

and this may be used to eliminate the variable x in equations (20) and (22) in favour of α. Performing the differentiation in (20) and using (22) to eliminate θ_S, we thus obtain after some simplification

$$\frac{d\beta}{d\alpha} = \frac{\beta}{\alpha}\left[\frac{2}{\sigma} - 1 + \frac{(\alpha - \beta)(\alpha e^{\alpha} - \sinh\alpha)}{\alpha\cosh\alpha - \sinh\alpha}\right] + \frac{2\eta e^{\alpha}}{\sigma\alpha^2}[(1 + \beta)\sinh\alpha - \alpha\cosh\alpha - \alpha\beta e^{-\alpha}]. \tag{24}$$

At $x = 0$, $\delta = \delta_T = 0$, and hence by equations (12) and (18), the end condition required for the integration of (24) is

$$\beta = 0 \quad \text{when} \quad \alpha = 0. \tag{25}$$

When β is known as a function of α, the dependence of θ_S on α can be determined by integration of equations (22) and (23), which gives

$$\log_e \frac{\theta_S(\alpha)}{\theta_S(0)} = -\frac{\eta}{\sigma}\int_0^{\alpha} \frac{1}{\beta\alpha^2}[(\alpha - 1)e^{2\alpha} + \alpha + 1]\mathrm{d}\alpha. \tag{26}$$

Inverting this expression, and restoring the original variables by means of equations (9), (14) and (15), the temperature $T_S(x)$ along the fibre is given by

$$T_S(x) = T_\infty + (T_0 - T_\infty)\exp(-\eta I/\sigma), \tag{27}$$

where

$$I = \int_0^{\alpha(x)} \frac{1}{\beta\alpha^2}[(\alpha - 1)e^{2\alpha} + \alpha + 1]\mathrm{d}\alpha. \tag{28}$$

Values of x corresponding to particular values of α may be calculated by means of equation (13).

As a footnote to this section it is worth mentioning that when $\beta(x)$ and $T_S(x)$ have been calculated, the local rate of heat transfer, $Q(x)$, per unit length of cylinder may also be deduced easily. It is readily verified that

$$Q(x) = 2\pi k[T_S(x) - T_\infty]/\beta(x). \tag{29}$$

4. DISCUSSION OF RESULTS

We note first that a little care must be taken in evaluating the integral I defined by equation (28) because the integrand is of an indeterminate form at the point $\alpha = 0$, $\beta = 0$. Furthermore, the right-hand side of the differential equation (24) is also singular at this point. Details of the computional method used are presented in the Appendix B.

Before calculating the results, it was necessary to assign suitable values to the Prandtl number σ, the kinematic viscosity ν and the parameter η [defined by equation (21)]. In the analysis to obtain the temperature distribution, given in the preceding section, these parameters were regarded as constants, but each of them is, of course, temperature dependent. It was hoped that at this stage mean values could be chosen in a consistent way which would lead to satisfactory agreement with the experimental results of Alderson *et al.* [10] and Arridge and Prior [11]. Experience showed that the final results were relatively insensitive to changes in σ, and as this parameter varies only slowly with temperature it was taken to have the value corresponding to the mid-point of whatever range of fibre temperatures was under consideration. The results were considerably influenced by variations in ν and η, and several ways of assigning suitable mean values were tried. The most successful procedure was the following.

Suppose that at the orifice the temperature of the fibre is T_0 and that it is required to determine the distance x_f at which the temperature is T_f. The mean values of ν and η with respect to temperature over the range (T_0, T_f) were calculated, using Tables of physical constants. These mean values were then used to obtain an estimate of x_f. By repeating the procedure for various values of T_f, a first approximation, $T_S^{(1)}(x)$, to the surface temperature distribution $T_S(x)$ was obtained.

To obtain a better approximation, the distribution $T_S^{(1)}(x)$ was used to calculate values of ν and η along the surface of the fibre, and this enabled mean values of ν and η with respect to distance x to be found. These mean values were then used to obtain a second approximation, $T_S^{(2)}(x)$, to $T_S(x)$. This sequence of approximations was continued, utilising at each stage the latest approximation to $T_S(x)$ to obtain approximations to the mean values with respect to x of ν and η, until no further noticeable change occurred. In practice, convergence was quite rapid and, with the exception of one case when a further iteration was required, it was found unnecessary to go beyond the third approximation.

The motivation for the above procedure is that the surface temperature distribution is likely to be strongly influenced by the average fluid properties near to the surface, since it is in this region that most of the heat transfer takes place. Average values with respect to distance along the surface, are, of course, likely to be more appropriate than average values with respect to the temperature range under consideration, and hence the latter were used only to initiate the approximations to $T_S(x)$. Ideally, some account should also be taken of spatial variations across the boundary layer, but investigation showed that very strong weighting towards surface values was necessary to bring the theoretical results close to the experimental ones. In view of the fact that any averaging procedure at this stage can be, at best, only an approximation there seems little justification for a more elaborate procedure than that described. It must also be borne in mind that there is an inherent error in the Pohlhausen method, since the assumed velocity and temperature profiles are not exact solutions of the basic differential equations, and this may well be at least as significant as that incurred in the process of averaging.

It is worth mentioning that, as an alternative to obtaining the mean values of ν and η with respect to x, the simpler procedure of using the values corresponding to the temperature at the point $\frac{1}{2}x_f$ was also tried. However, although this method sometimes led to satisfactory agreement with experimental results, it was found overall to be less reliable.

Alderson *et al.* [10] performed experiments on glass fibres and found that, within a quite substantial distance from the orifice, $\log [T_S(x) - T_\infty]$ decreases almost linearly with x. Figures 2–4 show the straight lines which they fitted to their data on three fibres of different radii drawn at different speeds. In each case, the orifice temperature (T_0) was about 1100°C and measurements were made up to the point at which the temperature had fallen to about 100°C. Using the method developed in this paper, theoretical calculations were made to determine the values of x corresponding to fibre temperatures of 100, 200, 300, 400 and 500°C, respectively. The points obtained are shown in the diagrams; Figs. 2 and 3 show only

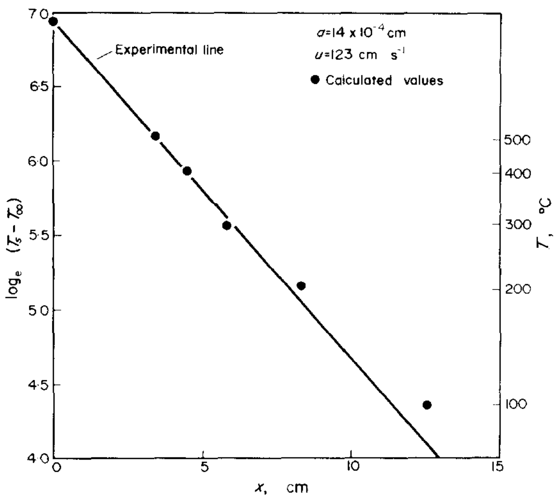

FIG. 2. Comparison between theoretical values and experimental line of Alderson *et al.* [10].

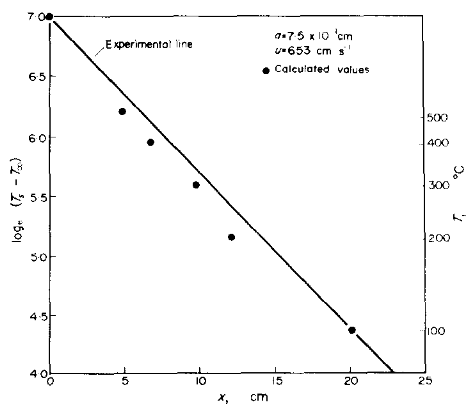

FIG. 3. Comparison between theoretical values and experimental line of Alderson *et al.* [10].

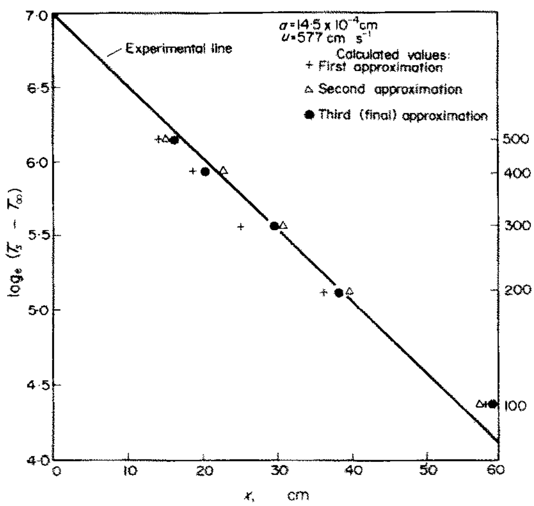

FIG. 4. Comparison between theoretical values (first, second and third approximations) and experimental line of Alderson *et al.* [10].

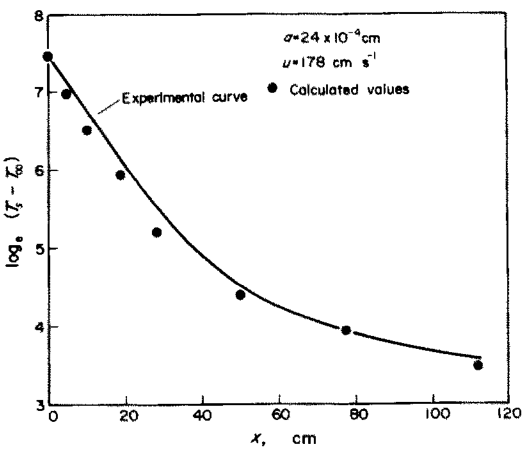

FIG. 5. Comparison between theoretical values and experimental curve of Arridge and Prior [11].

the results of the final approximations to $T_S(x)$, but Fig. 4 shows the first, second and third (final) approximation in order to give some indication of the extent of the differences between them in a typical case. It should be noted that sufficient tabulated data of some of the physical parameters involved could not be found and graphical interpolation was used where necessary. Due to errors incurred in this process, it was anticipated that some noticeable inaccuracies might be introduced into the calculations, and consequently the slight irregularity of the theoretical points which is evident in Figs. 2–4 was not unexpected. It is observed, however, that the calculated values lie reasonably close to the experimental lines. The most pronounced deviations occur in the second case (Fig. 3), where temperatures in the intermediate range of values of x are underestimated.

Arridge and Prior [11] also performed experiments on the cooling of silica fibres. They measured fibre temperatures over relatively longer ranges of x than did Alderson *et al.* [10], and obtained results in the region well beyond the range of validity of the approximately linear relationship between log $[T_S(x) - T_\infty]$ and x. In one case, they gave details of the measured temperature distribution, from an orifice value of 1750°C to a final value of about 55°C. Their experimental curve is shown in Fig. 5, together with some points predicted by the theory of this paper. As found in comparing with one of the experiments of Alderson *et al.* [10], there is a marked tendency to underestimate temperatures at intermediate values of x.

The discrepancies between the theoretical and experimental values are probably due largely to using average values of the fluid properties, some of which vary by as much as a factor of ten in the temperature ranges considered. As mentioned earlier, there may also be a significant contribution from the von Kármán–Pohlhausen approximation.

There are also obvious limitations to the model near the orifice which should be borne in mind. In the paper of Bourne and Elliston [1], no account was taken of variation in the fibre radius, which is quite marked near the orifice, and the same approximation has been adopted here. Furthermore, Griffin and Throne [13] found in experiments on a flat sheet issuing from a slot in a wall that the presence of the wall leads to a reduction in the heat transfer coefficient near the slot. A similar effect can be anticipated in the vicinity of the origin of a drawn fibre. On the other hand, Glicksman [14] has shown that near the orifice, radiation significantly enhances the heat transfer process. Using the appropriate formula in [14], it may be shown that this effect is of about the same order of magnitude as the reduction found by Griffin and Throne [13]. The net contribution to the error from these sources is thus probably quite small, particularly as the region where they are significant is only a small fraction of the total length of fibre considered.

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APPENDIX A

The purpose of this Appendix is to explain why the assumed temperature profile, defined by equations (16)–(18), is likely to serve adequately in calculating the surface temperature by the von Kármán–Pohlhausen method.

Consider a profile of the form

$$\frac{\theta}{\theta_S} = 1 - \frac{1}{\beta(x)}\left(\frac{y}{a} + p\frac{y^2}{a^2} + q\frac{y^3}{a^3}\right) \quad \text{for} \quad y \leqslant \delta_T(x) \tag{A.1}$$

and

$$\frac{\theta}{\theta_S} = 0 \quad \text{for} \quad y \geqslant \delta_T(x), \tag{A.2}$$

where p, q are constants and β is a function of x only. This satisfies the temperature boundary conditions (4) and (5). For good accuracy in calculating surface properties, it is also desirable that the chosen profile should adequately represent conditions near the surface. This may be achieved by choosing p and q so that, at $y = 0$, the thermal boundary layer equation (3) and its derivative with respect to y are satisfied.

Substituting (A.1) into the temperature equation (3) and using the definitions (14) and (15), we find that for the equation to be satisfied at $y = 0$,

$$U\frac{d\theta_S}{dx} = -\frac{\kappa\theta_S}{\beta a^2}(1 + 2p). \tag{A.3}$$

Eliminating θ_S by means of equation (22), it thus follows that

$$p = \eta - \tfrac{1}{2}. \tag{A.4}$$

However, in practice, $\eta = 0(10^{-3})$ and hence to a very good approximation we may take $p = -\tfrac{1}{2}$.

Using the continuity equation (1) and the velocity boundary conditions (4) and (5), the derivative at $y = 0$ of the temperature equation (3) reduces to

$$\left[\frac{\partial u}{\partial y}\frac{\partial T}{\partial x} + U\frac{\partial^2 T}{\partial x \partial y}\right]_{y=0} = \kappa\left[\frac{\partial^3 T}{\partial y^3} - \frac{1}{a}\frac{\partial^2 T}{\partial y^2} - \frac{1}{a^2}\frac{\partial T}{\partial y}\right]_{y=0} \tag{A.5}$$

Substituting the velocity profile (10) and the assumed temperature profile (A.1), and using equation (22) to eliminate θ_S, we find that

$$q = \frac{1}{3} - \frac{1}{3}\eta\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) - \frac{Ua^2}{6\kappa\beta}\frac{d\beta}{dx}. \tag{A.6}$$

Now, except within a distance of the order of 1 mm from the orifice ($x = 0$), the terms in α and β on the right-hand side of (A.6) are found to be very small indeed and hence we may take $q = \frac{1}{3}$ with negligible error. Thus for $y \leqslant \delta_T(x)$, we now have

$$\frac{\theta}{\theta_S} = 1 - \frac{1}{\beta(x)}\left[\frac{y}{a} - \frac{1}{2}\frac{y^2}{a^2} + \frac{1}{3}\frac{y^3}{a^3}\right] \tag{A.7}$$

to a good approximation. This profile is the same as the first four terms of the expansion in ascending powers of y of the logarithmic expression (16), and it follows that (16) is likely to have a good accuracy near the surface of the fibre ($y = 0$).

Finally, it should be noted that an important additional feature of the logarithmic form, which makes it preferable to a simple polynominal expression, is that it asymptotically correct when $x \to \infty$. For, in the limit when $x \to \infty$, the convection terms on the left-hand side of equation (3) vanish, and it is readily verified that the profile (16) satisfies the reduced equation identically.

APPENDIX B

At the point $\alpha = 0$, $\beta = 0$, the right-hand side of equation (24) is of an indeterminate form. To obtain β in this neighbourhood an expansion of the form

$$\beta = a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + \ldots \tag{B.1}$$

may be used. Substituting this into the differential equation (24) and comparing coefficients of α, α^2 and α^3 in the expansion of each side, we find that

$$a_1 = \tfrac{1}{3}(\sigma + 2)/\sigma, \tag{B.2}$$

$$a_2 = [(\sigma - 1)(\sigma + 2) + 6\eta]/9\sigma(\sigma + 1), \tag{B.3}$$

$$a_3 = [(\sigma - 1)(\sigma + 2)(3\sigma^2 - 4\sigma - 2) + 60(3\sigma^2 + \sigma + 2)\eta + 360\eta^2]/270(\sigma + 1)^2(3\sigma + 2). \tag{B.4}$$

To evaluate the integral I defined by equation (28), the series expansion (B.1) was substituted first, which on expanding in ascending powers of α gives

$$I = \frac{2}{3a_1}\left[1 + \left(1 - \frac{a_2}{a_1}\right)\alpha + \left(\frac{3}{5} - \frac{a_2}{a_1} - \frac{a_3}{a_1} + \frac{a_2^2}{a_1^2}\right)\alpha^2\right]. \tag{B.5}$$

This expression was used to evaluate I in the range $0 \leqslant \alpha \leqslant 0{\cdot}08$; outside this range, values of the integral were calculated by means of the trapezoidal rule.

The series expansion (B.1) was used to evaluate β in the range $0 \leqslant \alpha \leqslant 0{\cdot}08$. Using the Hamming method, forward integration of the differential equation (24) was then carried out, in steps of 0·02, to determine β up to the point where $\alpha = 7{\cdot}5$ (which is likely to be quite sufficient for most practical purposes).

Finally, values of x corresponding to particular values of α were found from equation (13), and hence the temperature distribution $T_S(x)$ was obtained.

The whole procedure was programmed and carried out by an I.C.T. 1907 computer.

LE REFROIDISSEMENT DE FIBRES DANS LE PROCESSUS DE FORMATION

Résumé—Dans les industries du verre et des polymères, des fibres sont fabriquées par extrusion du matériau chaud dans un orifice circulaire pour former un filament continu. Cet article concerne la détermination de la température de la fibre comme étant une fonction de la distance à l'orifice. On examine un simple exemple dans lequel la fibre est prise pour un cylindre continu circulaire infini sortant dans un fluide d'étendue infinie. Les équations de la couche limite en relation avec une équation contrôlant le taux de cession de chaleur de la fibre au fluide sont résolues par la méthode de Kármán–Pohlhausen.

Une difficulté importance dans ce problème est que les propriétés du fluide varient bien au-delà des hypothèses ceci à cause des grands gradients de température qui existent dans la pratique. Pour tirer compte de ceci une méthode de moyenne est développée qui conduit à un accord favorable avec les résultats expérimentaux.

DIE ABKÜHLUNG VON FASERN BEIM HERSTELLUNGSPROZESS

Zusammenfassung—In der Glas- und Kunststoffindustrie werden Fasern durch Herauspressen heissen Materials aus kreisförmigen Düsen erzeugt in Form eines endlosen Fadens. Hier soll die Fadentemperatur als Funktion vom Abstand von der Düse bestimmt werden.

Im Modell wird der Faden als endloser Kreiszylinder angesehen, der kontinuierlich in ein unendlich grosses Bad taucht. Nach der Kármán–Pohlhausen-Methode sind die Grenzschichtgleichungen zusammen mit einer Gleichung für den Wärmetransport vom Faden an die Flüssigkeit gelöst.

Eine beachtenswerte Schwierigkeit liegt hier darin, dass sich die Flüssigkeitseigenschaften im interessierenden Bereich stark ändern, wegen der praktisch vorkommenden grossen Temperaturgradienten. Um diese Schwierigkeit zu umgehen, wird eine Methode zur Mittelwertbildung vorgeschlagen, die ziemlich gute Übereinstimmung mit Versuchswerten liefert.

ОХЛАЖДЕНИЕ ВОЛОКНА В ТЕХНОЛОГИЧЕСКОМ ПРОЦЕССЕ

Аннотация—В производственных условиях непрерывные волокна стекла и полимеров получают путем продавливания нагретого материала через круглые отверстия. В данной работе приведён расчёт температуры волокна как функции расстояния от отверстия. Принята простая расчётная модоль, по которой волокно рассматривается как сплошной непрерывный бесконечный круглый цилиндр, погружающийся в безграничную жидкость. Уравнения пограничного слоя решаются методом Кармана-Польгаузена совместно с уравнением теплового баланса, определяющим интенсивность теплоотдачи от волокна в жидкость. Наибольшая трудность в решении этой задачи состоит в необходимости учёта сильной переменности свойств жидкости во всей исследуемой области под воздействием больших температурных градиентов, имеющих место в реальном процессе. В работе прецлагается метод осреднения, обеспечивающий довольно хорошее согласование расчёта с экспериментальными результатами.